OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4756/01

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

MONDAY 25 JUNE 2018: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper supplied by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4756/01 sent with the standard paper
MEI Examination Formulae and Tables (MF2) sent with the standard paper

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (RW) 172338/1

8028743062

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (54 marks)

1 (a) The polar equation of a curve is $r = a\sin^2\theta\cos\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Find the value of θ for which the curve has the maximum x-coordinate. [3]

(ii) Prove that the maximum y-coordinate on the curve is $\frac{3\sqrt{3}}{16}a$ and state the value of θ at which this is attained. [4]

(b) (i) Sketch the graph of $y = \arcsin x$ for $-1 \leqslant x \leqslant 1$. [1]

(ii) Prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$. [4]

(iii) Using integration by parts and a suitable substitution, show that

$$\int_0^1 x^2 \arcsin x \,\mathrm{d}x = \frac{3\pi - 4}{18}.$$ [6]

2 (a) (i) Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6\tan^2\theta + \tan^4\theta}{4\tan\theta(1 - \tan^2\theta)}.$$ [5]

(ii) Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometrical form. [4]

(b) The vertices of a square with sides of length 1 unit lie on the axes of an Argand diagram. The vertices represent the complex numbers z_1, z_2, z_3 and z_4 and the midpoints of the sides of the square represent the complex numbers z_5, z_6, z_7 and z_8.

(i) Express z_5, z_6, z_7 and z_8 in modulus-argument form, and hence determine a polynomial equation of degree 4, with integer coefficients, whose roots are z_5, z_6, z_7 and z_8. [4]

Let $\mathrm{P}(z) = 0$ be a polynomial equation of degree 8, with integer coefficients, whose roots are z_1, z_2, z_3, z_4, z_5, z_6, z_7 and z_8.

(ii) Explain why $\mathrm{P}(z)$ cannot be of the form $az^8 + b$ where a and b are integers. [1]

(iii) Find $\mathrm{P}(z)$. [4]

3 (i) Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$. [5]

The matrix $\mathbf{M}$ has eigenvalues 1, 2 and 3. The corresponding eigenvectors are $\begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ respectively.

(ii) Write down the matrix $\mathbf{P}$ such that $\mathbf{M} = \mathbf{PDP}^{-1}$ where $\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. [2]

(iii) Hence find $\mathbf{M}$. [5]

(iv) Find constants a, b and c such that $\mathbf{M}^{-1} = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. [6]

SECTION B (18 marks)

4 (i) Prove, using definitions in terms of exponential functions, that

$$\cosh 2A = 1 + 2\sinh^2 A. \quad [3]$$

(ii) Find $\int \sinh^2 x \, \mathrm{d}x$. [3]

(iii) Let $z = \operatorname{arsinh}(1)$. Form an equation involving z and solve it to find the exact value of arsinh(1) in logarithmic form. [4]

(iv) Using a substitution of the form $ax = b\sinh u$, find the exact value of

$$\int_0^{\frac{2}{3}} \frac{x^2}{\sqrt{4 + 9x^2}} \, \mathrm{d}x,$$

giving your answer in the form $p(q - \ln r)$, where p, q and r are constants. [8]

END OF QUESTION PAPER